

# Effect of FCNC mediated $Z$ boson on lepton flavor violating decays

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## Abstract

We study the three body lepton flavor violating (LFV) decays  $\mu^- \rightarrow e^- e^+ e^-$ ,  $\tau^- \rightarrow l_i^- l_j^+ l_j^-$  and the semileptonic decay  $\tau \rightarrow \mu \phi$  in the flavor changing neutral current (FCNC) mediated  $Z$  boson model. We also calculate the branching ratios for LFV leptonic  $B$  decays,  $B_{d,s} \rightarrow \mu e$ ,  $B_{d,s} \rightarrow \tau e$ ,  $B_{d,s} \rightarrow \tau \mu$  and the conversion of muon to electron in Ti nucleus. The new physics parameter space is constrained by using the experimental limits on  $\mu^- \rightarrow e^- e^+ e^-$  and  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ . We find that the branching ratios for  $\tau \rightarrow eee$  and  $\tau \rightarrow \mu \phi$  processes could be as large as  $\sim \mathcal{O}(10^{-8})$  and  $\text{Br}(B_{d,s} \rightarrow \tau \mu, \tau e) \sim \mathcal{O}(10^{-10})$ . For other LFV  $B$  decays the branching ratios are found to be too small to be observed in the near future.

PACS numbers: 13.35.Bv, 13.35.Dx, 13.20.He, 12.60.Cn

## I. INTRODUCTION

It is very well known that in the standard model (SM) of electroweak interactions the generation lepton number is exactly conserved. However, the observation of neutrino oscillation implies that family lepton number must be violated. The neutrino oscillation is due to a mismatch between the weak and mass eigenstates of neutrinos and this mismatch causes mixing between different generation of leptons in the charged current interaction of the  $W$  boson. Due to the violation of family lepton number, flavour changing neutral current (FCNC) processes in the lepton sector could in principle occur, analogous to the quark sector. Some examples of FCNC transitions in the lepton sector would be  $l_i \rightarrow l_j \gamma$ ,  $l_i \rightarrow l_j l_k \bar{l}_k$ ,  $B \rightarrow l_i \bar{l}_j$  and  $B \rightarrow X_s l_i \bar{l}_j$  (where  $l$  is any charged lepton) etc. Although there is no direct conclusive experimental evidence for such processes that have been observed so far, but there exist severe constraints on some of these LFV decay modes [1]. It should be noted that FCNC transitions in the lepton sector that are solely due to mixing in the charged current interaction with the usual left handed  $W$  boson and light neutrinos are extremely small because they are suppressed by powers of  $m_\nu^2/M_W^2$ . In particular the branching ratio for  $\mu \rightarrow e \gamma$  in the SM amounts to at most  $10^{-54}$  [2], to be compared with the present experimental upper bound  $1.2 \times 10^{-11}$  [1]. Therefore any observation of lepton flavour violation (LFV) in the foreseeable future would be an unambiguous signal of new physics beyond the SM. One way to increase the FCNC interactions in the lepton sector is to introduce heavy neutrinos so that the suppression factor  $m_\nu^2/M_W^2$  is not affected. This can be done for example by introducing heavy fourth generation [3]. If one insists on having just three left handed neutrinos one needs to give the right handed neutrinos heavy Majorana masses. As a consequence, these observations often put severe constraints on the parameter space of new physics models in which heavy leptons are present. Moreover these decays being unaffected by hadronic uncertainties, allow for a clear distinction between different new physics scenarios, in particular when several branching ratios are considered simultaneously.

Many models for physics beyond the standard model predicts lepton flavor violating decays of charged leptons at a level which may become observable very soon [4]. The LFV tau decays are analyzed in a model independent way in Ref. [5]. In the present paper we investigate the LFV  $\mu$  and  $\tau$  decays e.g.,  $\mu, \tau \rightarrow l_i l_j \bar{l}_j$ ,  $\mu \rightarrow e \nu_e \bar{\nu}_\mu$ ,  $\tau \rightarrow \mu \phi$  and  $B_{d,s} \rightarrow l_i \bar{l}_j$  in a model where  $Z$  mediated flavor changing neutral current (FCNC) transitions occur at

the tree level. It is well known that FCNC coupling of the  $Z$  boson can be generated at the tree level in various exotic scenarios. Two popular examples discussed in the literature are the models with an extra  $U(1)$  symmetry [6] and those with the addition of non-sequential generation of quarks or leptons [7]. In the case of extra  $U(1)$  symmetry the FCNC couplings of the  $Z$  boson are induced by  $Z - Z'$  mixing, provided the SM quarks/leptons have family non-universal charges under the new  $U(1)$  group. In the second case, adding a different number of quarks or leptons, the pseudo mixing matrix needed to diagonalize the charged currents is no longer unitary and this leads to tree level FCNC couplings. It should be noted that, recently, there has been renewed interests shown in the literature concerning the non-universal  $Z$  induced new physics [8] in the quark sector.

Here we will follow the second approach [7] to analyze some FCNC induced rare LFV decays. We consider the presence of an additional vector like sterile neutrino, which could mix with the SM three neutrinos resulting a  $4 \times 4$  mixing matrix  $V$  for the neutrinos. Due to such mixing however the charged current interactions remain unchanged except that the SM PMNS mixing matrix  $V_{PMNS}$  is now the  $3 \times 4$  upper sub-matrix of  $V$ . However, the distinctive feature of this model is that the FCNC interaction enters the neutral current Lagrangian of the left handed neutrinos as

$$\mathcal{L}_Z = \frac{g}{2 \cos \theta_W} \left[ \bar{l}_{Li} \gamma^\mu l_{Li} - \bar{\nu}_{L\alpha} U_{\alpha\beta} \gamma^\mu \nu_{L\beta} - 2 \sin^2 \theta_W J_{em}^\mu \right] Z_\mu , \quad (1)$$

with

$$U_{\alpha\beta} = \sum_{i=\nu_e, \nu_\mu, \nu_\tau, \nu_s} V_{\alpha i}^\dagger V_{i\beta} = \delta_{\alpha\beta} - V_{4\alpha}^* V_{4\beta} , \quad (2)$$

where  $U$  is the neutral current mixing matrix for the neutrino sector, which is given above. As  $V$  is not unitary,  $U \neq \mathbf{1}$ . In particular the non-diagonal elements do not vanish.

$$U_{\alpha\beta} = -V_{4\alpha}^* V_{4\beta} \neq 0 \quad \text{for } \alpha \neq \beta . \quad (3)$$

Since the various  $U_{\alpha\beta}$  are non vanishing, they would signal new physics and the presence of FCNC at the tree level and this can substantially mediate many low energy LFV processes. In this paper we consider the impact of FCNC mediated  $Z$  boson couplings on several LFV decays of  $\mu$ ,  $\tau$  and  $B_{d,s}$  mesons. There is also huge experimental efforts going on to look for any possible signals of LFV decays. For instance, the recent commencement of the MEG experiment [9], which will probe  $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-13}$  two orders magnitude

beyond the current limit. Also the  $B$  factories, Belle and Babar have searched for the decay modes  $\tau \rightarrow l_i l_j \bar{l}_j$  with upper limits in the range  $\text{Br}(\tau \rightarrow l_i l_j \bar{l}_j) < (2 - 8) \times 10^{-8}$  [10]. Searches for  $\tau \rightarrow \mu \mu \bar{\mu}$  can be performed at the Large Hadron Collider where  $\tau$  leptons are copiously produced from the decays of  $W$ ,  $Z$ ,  $B$  and  $D$ , with anticipated sensitivities  $\text{Br}(\tau \rightarrow \mu \mu \bar{\mu}) \sim 10^{-8}$  [11].

The outline of the paper is as follows. The LFV decays  $\mu^- \rightarrow e^- e^+ e^-$  and  $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$  are presented in section II and  $B_{d,s} \rightarrow \mu^\pm e^\mp$  in section III. The LFV tau decays are discussed in section IV. In section V we discuss the  $\mu - e$  conversion in Titanium and  $\mu \rightarrow e \gamma$  process. section VI contains the summary and conclusion.

## II. DECAY RATES FOR $\mu^- \rightarrow e^- e^+ e^-$ AND $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$

Let us now consider the decay mode  $\mu^-(p) \rightarrow e^-(p_1) + e^-(p_2) + e^+(p_3)$  which has a strict bound of  $10^{-12}$ . Since this mode is highly suppressed in the SM with a branching ratio  $\text{Br}(\mu \rightarrow eee) \simeq 10^{-17}$  [12], where the presence of a right-handed neutrino is assumed. Therefore, in our analysis we will not include the SM contributions to this process. In the FCNC mediated  $Z$  boson model, it occurs at the tree level due to the presence of FCNC coupling of the  $Z$  boson. Hence it is expected that in such a model the branching ratio can be substantially enhanced and it could be possible that this mode can be probed at current and forthcoming experiments. The Feynman diagram for this process is shown in Figure 1, where the blob represents the tree level FCNC coupling of  $Z$  boson (lepton flavor violating coupling). Thus, one can obtain the amplitude for this process as

$$\frac{G_F}{\sqrt{2}} U_{\mu e} \left( [\bar{e}(p_1) \gamma^\mu (1 - \gamma_5) \mu(p)] [\bar{e}(p_2) \gamma_\mu (C_V - C_A \gamma_5) e(p_3)] + (p_1 \leftrightarrow p_2) \right), \quad (4)$$

where  $p_i$ 's represent the momenta of different particles involved in this process.  $C_V$  and  $C_A$  are the vector and axial-vector couplings of the  $Z$  boson to electron-positron pair, with values  $C_V = -1/2 + 2 \sin^2 \theta_W$  and  $C_A = -1/2$ .  $U_{\mu e}$  is the lepton flavor violating FCNC coupling at the  $\mu e Z$  vertex.

Thus, from the above amplitude, one can obtain the decay rate after doing a simple calculation and three body phase space integration, as

$$\Gamma(\mu^- \rightarrow e^- + e^+ + e^-) = \frac{G_F^2}{384 \pi^3} |U_{\mu e}|^2 (C_A - C_V)^2 m_\mu^5, \quad (5)$$

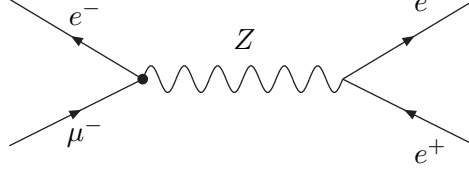


FIG. 1: Feynman diagram for  $\mu^- \rightarrow e^- e^+ e^-$  in the model with FCNC mediated  $Z$  boson, where the blob represents the tree level FCNC coupling (lepton flavor changing vertex) of  $Z$  boson.

where we have neglected the electron mass. Now using the experimental upper limit of the branching ratio of this mode  $\text{Br}(\mu^- \rightarrow e^- e^+ e^-) < 10^{-12}$  [1], we obtain the upper limit on the lepton flavor violating coupling  $U_{\mu e}$  as

$$|U_{\mu e}| < 3.05 \times 10^{-6}, \quad (6)$$

where we have used the mass and lifetime of muon from [1] and  $\sin^2 \theta_W = 0.231$ .

Next we consider another LFV  $\mu$  decay,  $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$ , which violates  $L_e$  and  $L_\mu$  by two units each and hence it is highly suppressed in the SM. However, in the model with FCNC mediated  $Z$  boson this process can occur at the tree level with two LFV vertices and the corresponding Feynman diagram is shown in Figure-2 where  $l_1$  and  $l_2$  denote  $\mu$  and  $e$ ,  $f_1$  and  $f_2$  as  $\nu_e$  and  $\nu_\mu$ . The amplitude for this process is given as

$$\mathcal{M}(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu) = \frac{G_F}{2\sqrt{2}} |U_{\mu e}|^2 [\bar{e} \gamma^\mu (1 - \gamma_5) \mu] [\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \nu_e]. \quad (7)$$

The corresponding decay rate is found to be (neglecting electron mass)

$$\Gamma = \frac{G_F^2}{384\pi^3} |U_{\mu e}|^4 m_\mu^5. \quad (8)$$

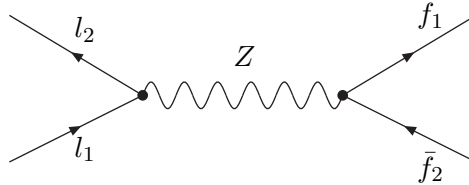


FIG. 2: Feynman diagram for  $l_1 \rightarrow l_2 f_1 \bar{f}_2$  in the model with FCNC mediated  $Z$  boson, where the blobs represent the tree level FCNC vertices.

Now varying the value of  $|U_{\mu e}|$  between  $0 \leq |U_{\mu e}| \leq 3.05 \times 10^{-6}$ , extracted from  $\mu \rightarrow eee$  process, we show in Figure-3 the correlation plot between the branching ratios of  $\mu^- \rightarrow e^- e^+ e^-$  and  $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$  processes. From the figure it can be seen that the maximum value of branching ratio that can be accommodated for the process  $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$  in the extra  $Z$  boson model considered here as

$$\text{Br}(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu) < 4.36 \times 10^{-23}, \quad (9)$$

which is well below the present experimental upper limit [1]

$$\text{Br}(\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu) < 1.2 \times 10^{-2}. \quad (10)$$

Since the branching ratio is well below the sensitivities of the present and upcoming experiments there is no chance to observe this mode in the near future. Since the final neutrinos are difficult to detect it is useful to consider the channel  $\mu^- \rightarrow e^- \nu_\alpha \bar{\nu}_\alpha$ , which involves only one FCNC  $Z - e - \mu$  coupling in contrast to the previous case where there are two such couplings. The decay rate for this process is given as

$$\Gamma(\mu \rightarrow e \nu_\alpha \bar{\nu}_\alpha) = \frac{G_F^2}{384\pi^3} |U_{\mu e}|^2 m_\mu^5. \quad (11)$$

Using the bound on  $|U_{\mu e}|$  and summing over all neutrino flavors, we obtain the branching ratio as

$$\text{Br}(\mu \rightarrow e \nu \bar{\nu}) < 1.4 \times 10^{-11}, \quad (12)$$

which could be accessible in the future high sensitivity experiments.

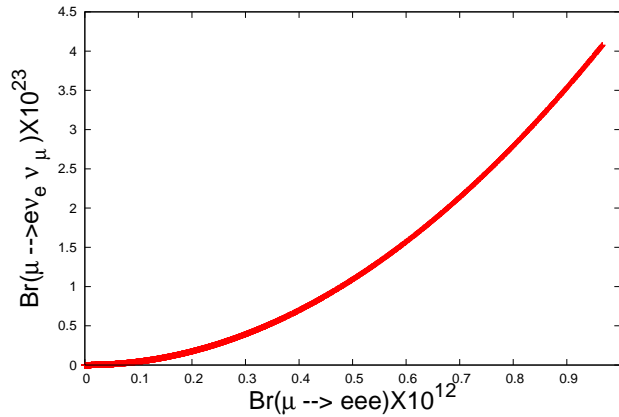


FIG. 3: Correlation plot between the branching ratios of  $\mu^- \rightarrow e^- e^+ e^-$  and  $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$  processes.

### III. $B_{d,s} \rightarrow \mu^\pm e^\mp$

Now we consider the lepton flavour violating  $B$  meson decays  $B_{d,s} \rightarrow l_i \bar{l}_j$ . Here we will consider only the decay mode  $B_s \rightarrow \mu^\pm e^\mp$ , since the same formula will hold good for all the above mentioned processes by replacing appropriate particle masses. The corresponding Feynman diagram will be similar to the figure-2, with two FCNC vertices one for the quark and one for the lepton parts i.e., replacing  $l_1$  and  $l_2$  by  $b$  and  $s$  and  $(f_i, f_2)$  by  $(\mu, e)$ . The effective Hamiltonian describing this process is given by

$$\mathcal{H}_{eff} = \frac{G_F}{2\sqrt{2}} U_{bs} U_{\mu e} (\bar{s} \gamma^\mu (1 - \gamma_5) b) (\bar{\mu} \gamma_\mu (1 - \gamma_5) e), \quad (13)$$

where  $U_{bs}$  represents the FCNC coupling of the quark sector. To evaluate the transition amplitude, we use the following matrix element of the quark current between the initial  $B_s$  meson and *vacuum* as

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s(p_B) \rangle = i f_{B_s} p_B^\mu. \quad (14)$$

Thus, we obtain the amplitude for  $B_s \rightarrow \mu e$  process as

$$\mathcal{M}(B_s \rightarrow \mu e) = \frac{-i G_F}{2\sqrt{2}} U_{bs} U_{\mu e} m_\mu f_{B_s} [\bar{\mu} (1 - \gamma_5) e], \quad (15)$$

and the corresponding decay width as

$$\Gamma(B_s \rightarrow \mu^\pm e^\mp) = \frac{G_F^2}{16\pi} m_\mu^2 m_{B_s} |U_{sb} U_{\mu e}|^2 f_{B_s}^2 \left(1 - \frac{m_\mu^2}{m_B^2}\right)^2. \quad (16)$$

For numerical estimation we use the particle masses and lifetime from [1], the  $B_s$  meson decay constant as  $f_{B_s} = 0.24$  GeV. Now using the value of  $|U_{bs}|$  as  $0 \leq |U_{bs}| \leq 0.005$  [13], which is extracted from the mass difference of  $B_s - \bar{B}_s$  system and varying  $|U_{\mu e}|$  between  $0 \leq |U_{\mu e}| < 3.05 \times 10^{-6}$ , we show in figure-4 the correlation plot between the branching ratios of  $\mu^- \rightarrow e^- e^+ e^-$  and  $B_s \rightarrow \mu^\pm e^\mp$ . From the figure, one can obtain the upper limit of the branching ratio as

$$\text{Br}(B_s \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-18}. \quad (17)$$

Similarly for  $B_d \rightarrow \mu^\pm e^\mp$  process using the value of FCNC  $Zbd$  coupling as  $|U_{bd}| \leq 10^{-3}$  [14] and  $f_{B_d} = 0.22$  GeV, we obtain the corresponding branching ratio upper limit as

$$\text{Br}(B_d \rightarrow \mu^\pm e^\mp) < 1.7 \times 10^{-19}. \quad (18)$$

Since these rates are also highly suppressed there is also no possibility to observe these modes in the near future.

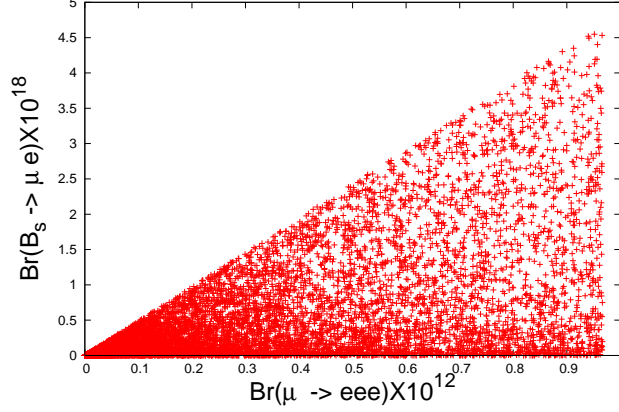


FIG. 4: Correlation plot between the branching ratios of  $\mu^- \rightarrow e^-e^+e^-$  and  $B_s \rightarrow \mu^\pm e^\mp$  processes.

#### IV. LEPTON FLAVOUR VIOLATING $\tau$ DECAYS

Now we consider the lepton flavour violating  $\tau$  decays. The LFV decays  $\tau \rightarrow lll$  are in analogy of  $\mu \rightarrow eee$  and provide sensitive probe of the lepton flavor violating couplings  $U_{\tau l}$  in the FCNC mediated  $Z$  boson model. More observation of such decays would constitute a spectacular signal of physics beyond the SM. There are six distinct decays for  $\tau^- \rightarrow lll$ :  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ ,  $\tau^- \rightarrow e^- e^+ e^-$ ,  $\tau^- \rightarrow \mu^- \mu^+ e^-$ ,  $\tau^- \rightarrow \mu^- \mu^- e^+$ ,  $\tau^- \rightarrow e^- e^+ \mu^-$ ,  $\tau^- \rightarrow e^- e^- \mu^+$ . Searches for all six decays have been performed by BABAR and Belle [10] and upper limits of the order  $\text{Br}(\tau \rightarrow lll) \sim \mathcal{O}(10^{-8})$  are obtained. Although these limits are several orders of magnitude weaker than the bound  $\text{Br}(\mu \rightarrow eee) < 10^{-12}$ , they have the virtue of constraining many combinations of  $U_{ij}$ . Moreover greater sensitivity to  $\text{Br}(\tau \rightarrow lll)$  is expected from forthcoming experiments.

Here we will only focus on  $\tau \rightarrow l_i l_j \bar{l}_j$  processes having only one LFV vertex. The corresponding Feynman diagram will be analogous to that of Figure-1 describing  $\mu \rightarrow eee$  process. First we will focus on  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$  and  $\tau^- \rightarrow e^- e^+ e^-$  which will allow us to obtain the constraint on  $U_{\tau\mu}$  and  $U_{\tau e}$ . The branching ratios for such processes will have the same form as Eq. (8) with the muon mass replaced by the *tau* mass and taking into account the appropriate LFV coupling  $U_{\tau l}$ . Using the upper limits of the branching ratios from [1], we obtain the limits on the LFV couplings as follows.

$$\begin{aligned} \text{Br}(\tau^- \rightarrow e^- e^+ e^-) < 3.6 \times 10^{-8}, &\Rightarrow |U_{\tau e}| < 1.37 \times 10^{-3} \\ \text{Br}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 3.2 \times 10^{-8} &\Rightarrow |U_{\tau\mu}| < 1.295 \times 10^{-3}. \end{aligned} \quad (19)$$



These constraints can be used to predict the branching ratios of LFV  $B$  decays  $B_{d,s} \rightarrow \tau\mu, \tau e$ . Now using the above constraint on  $U_{\tau\mu}$ , in Figure-5 we show the correlation plot between the branching ratios of  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$  and  $B_s \rightarrow \tau\mu$ . The branching ratio upper limit for this process is found to be

$$\text{Br}(B_s \rightarrow \tau\mu) = 1.9 \times 10^{-10}. \quad (20)$$

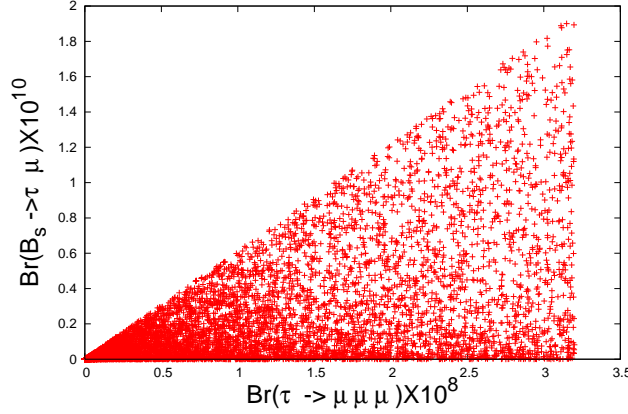


FIG. 5: Correlation plot between the branching ratios of  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$  and  $B_s \rightarrow \tau\mu$  processes.

#### A. $\tau^- \rightarrow e^- \mu^+ \mu^-$

Next we consider the decay process  $\tau^- \rightarrow e^- \mu^+ \mu^-$ . The Feynman diagram for this process is similar to Fig-1 with one LFV coupling. Although this process can also have contribution with two LFV couplings but such contribution is highly suppressed. The amplitude for this process is given as

$$\mathcal{M}(\tau^- \rightarrow e^- \mu^+ \mu^-) = \frac{G_F}{\sqrt{2}} U_{\tau e} [\bar{e}(p_1) \gamma^\mu (1 - \gamma_5) \tau(p)] [\bar{\mu}(p_2) \gamma_\mu (C_V - C_A \gamma_5) \mu(p_3)], \quad (21)$$

and the corresponding branching ratio as

$$\Gamma(\tau^- \rightarrow e^- \mu^+ \mu^-) = \frac{G_F^2}{384\pi^3} |U_{\tau e}|^2 m_\tau^5 (C_V^2 + C_A^2). \quad (22)$$

Using the branching ratio  $\text{Br}(\tau^- \rightarrow e^- e^+ e^-) \leq 3.7 \times 10^{-8}$  we obtain the constraint on  $|U_{\tau e}|$  as

$$|U_{\tau e}| < 1.28 \times 10^{-3}. \quad (23)$$

## B. $\tau \rightarrow \mu\phi$

Here we consider the LFV violating semileptonic decay mode of  $\tau$  lepton  $\tau \rightarrow \mu\phi$  and the corresponding Feynman diagram is analogous to Fig-1. The amplitude for this process

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} U_{\tau\mu} (\bar{\mu}\gamma^\mu(1 - \gamma_5)\tau) (\bar{s}\gamma_\mu(C_V^s - C_A^s\gamma_5)s), \quad (24)$$

where  $C_{V,A}^s$  are the vector/axial vector couplings of  $s\bar{s}$  quarks to the  $Z$  boson. Now to evaluate the transition amplitude, we use the following matrix element

$$\langle\phi(p', \epsilon)|\bar{s}\gamma^\mu s|0\rangle = f_\phi m_\phi \epsilon^\mu \quad (25)$$

where  $f_\phi$  is the decay constant of  $\phi$  meson and  $\epsilon$  being its polarization vector. With this we obtain the amplitude for this process as

$$\mathcal{M}(\tau \rightarrow \mu\phi) = \frac{G_F}{\sqrt{2}} U_{\mu\tau} C_V^s f_\phi m_\phi (\bar{\mu}\gamma^\mu(1 - \gamma_5)\tau) \epsilon_\mu. \quad (26)$$

Neglecting the muon mass, the corresponding decay rate is found to be

$$\Gamma = \frac{G_F^2}{16\pi} |U_{\tau\mu}|^2 (C_V^s)^2 f_\phi^2 m_\phi^2 m_\tau \left(1 - \frac{m_\phi^2}{m_\tau^2}\right)^2 \left(2 + \frac{m_\tau^2}{m_\phi^2}\right) \quad (27)$$

Now using  $\phi$  meson decay constant  $f_\phi = 0.231$  GeV, and varying  $U_{\tau\mu}$  between  $0 \leq |U_{\tau\mu}| \leq 1.295 \times 10^{-3}$ , we present in Figure-6, the correlation plot between  $\tau \rightarrow \mu\mu\mu$  and  $\tau \rightarrow \mu\phi$ . Since the upper limit of  $\text{Br}(\tau \rightarrow \mu\phi) \sim \mathcal{O}(10^{-8})$ , there is possibility that this mode could be observable in the near future.

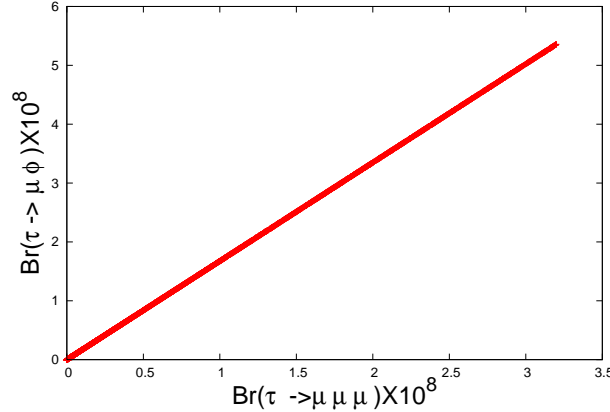


FIG. 6: Correlation plot between the branching ratios of  $\tau^- \rightarrow \mu^- \mu^+ \mu^+$  and  $\tau \rightarrow \mu\phi$  processes.

## V. $\mu - e$ CONVERSION IN NUCLEI AND $\mu \rightarrow e\gamma$ PROCESS

Now we will consider another example of lepton flavour violating process i.e., conversion of muon into electron in nuclei, where very stringent experimental upper bounds exist. In particular, the experimental upper bound on  $\mu - e$  conversion in  ${}^{48}_{22}\text{Ti}$  is [15]

$$R(\mu\text{Ti} \rightarrow e\text{Ti}) < 4.3 \times 10^{-12}, \quad (28)$$

and the dedicated J-PARC experiment PRISM/PRIME [16] should reach a sensitivity of  $\mathcal{O}(10^{-18})$ . A very detailed calculation of  $\mu - e$  conversion rate in various nuclei has been performed in [17]. Following Ref. [18], one can obtain the conversion rate in nuclei (normalized to the total nuclear muon rate  $\Gamma_{\text{capture}}$ ) in the FCNC mediated  $Z$  boson model as

$$R(\mu\text{Ti} \rightarrow e\text{Ti}) = \frac{G_F^2 \alpha^3 m_\mu^5}{4\pi^2} \frac{Z_{\text{eff}}^4}{Z} \frac{|U_{\mu e}|^2}{\Gamma_{\text{capture}}} |F(q^2)|^2 Q_W^2 \quad (29)$$

where

$$Q_W = (2Z + N)C_V^u + (Z + 2N)C_V^d, \quad (30)$$

is the coherent nuclear charge associated with the vector current of the nucleus, which gives an enhanced contribution to the coherent nuclear transition.  $C_V^u$  and  $C_V^d$  are the vector couplings of up and down quarks to the  $Z$  boson given as

$$C_V^u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad C_V^d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W. \quad (31)$$

$F(q^2)$  is the nuclear form factor and for  ${}^{48}_{22}\text{Ti}$  its value is found to be  $F(q^2 = -m_\mu^2) \simeq 0.54$ , and  $Z_{\text{eff}} \simeq 17.6$  [18]. For  $\Gamma_{\text{capture}}$ , we use its experimental value  $\Gamma_{\text{capture}} = (2.590 \pm 0.012) \times 10^6 \text{sec}^{-1}$  [19]. The variation of  $R(\mu\text{Ti} \rightarrow e\text{Ti})$  with  $|U_{\mu e}|$  is shown in Figure-7 (red curve). The horizontal line represents the experimental upper limit. From the figure it can be seen that much stronger constraint on  $|U_{\mu e}|$ , i.e.,  $|U_{\mu e}| < 1.05 \times 10^{-6}$ , can be obtained from the  $\mu - e$  conversion rate. This bound on  $|U_{\mu e}|$  is about three times stronger than the constraint obtained from  $\mu \rightarrow eee$  process.

Another well-known example of lepton flavour violating process is  $\mu \rightarrow e\gamma$ . However, this process occurs at one loop level in the FCNC mediated  $Z$  boson model as shown in Figure-8, where the internal fermion line  $l_i = (e, \mu, \tau)$ . Here we will consider the internal fermion lines to be either  $\mu$  or  $e$  so that we will have only one FCNC  $Z\mu e$  vertex. However, we will neglect

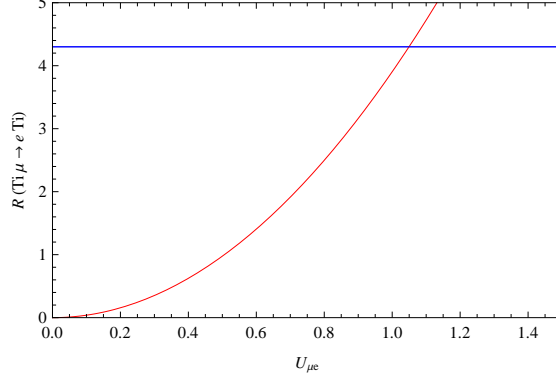


FIG. 7: Variation of  $\mu - e$  conversion rate  $R(\mu Ti \rightarrow e Ti)$  (in units of  $10^{-12}$ ) with  $|U_{\mu e}|$  (red curve). The horizontal blue line represents the experimental upper limit.

the contribution coming from internal electron line as it is proportional to  $(m_e/m_\mu)$ . Thus, we obtain the decay rate for  $\mu \rightarrow e\gamma$  as

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha G_F^2 m_\mu^5}{32\pi^4} |U_{\mu e}|^2 (C_V^\mu - C_A^\mu)^2. \quad (32)$$

Now using the bound on  $|U_{\mu e}| < 10^{-6}$ , we obtain the branching ratio

$$\text{Br}(\mu \rightarrow e\gamma) < 3 \times 10^{-15}, \quad (33)$$

which is well below the present experimental upper limit [1]

$$\text{Br}(\mu \rightarrow e\gamma)_{\text{expt}} < 1.2 \times 10^{-11}. \quad (34)$$

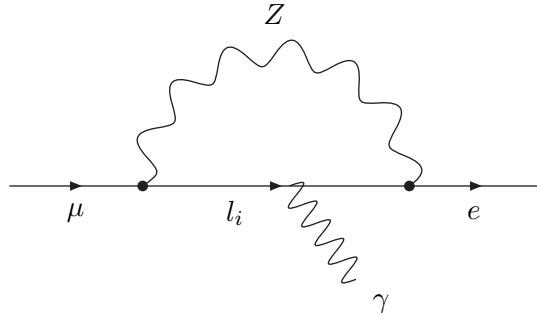


FIG. 8: Feynman diagram for  $\mu \rightarrow e\gamma$  in the model with FCNC mediated  $Z$  boson.

| Decay process                               | Predicted Br<br>(maximal value) | Experimental<br>upper limits [1] |
|---|---------------------------------|----------------------------------|
| $\mu^- \rightarrow e^- e^+ e^-$             | $1.0 \times 10^{-12}$           | $1.0 \times 10^{-12}$            |
| $\tau^- \rightarrow e^- \mu^+ \mu^-$        | $3.7 \times 10^{-8}$            | $3.7 \times 10^{-8}$             |
| $\tau^- \rightarrow \mu^- \mu^+ \mu^-$      | $3.2 \times 10^{-8}$            | $3.2 \times 10^{-8}$             |
| $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$ | $4.36 \times 10^{-23}$          | $1.2 \times 10^{-2}$             |
| $\tau^- \rightarrow e^- e^+ e^-$            | $3.1 \times 10^{-8}$            | $3.7 \times 10^{-7}$             |
| $\tau^- \rightarrow \mu^- \phi$             | $5.4 \times 10^{-8}$            | $1.3 \times 10^{-7}$             |
| $B_d \rightarrow e^\pm \mu^\mp$             | $1.7 \times 10^{-19}$           | $6.4 \times 10^{-8}$             |
| $B_s \rightarrow e^\pm \mu^\mp$             | $4.7 \times 10^{-18}$           | $2.0 \times 10^{-7}$             |
| $B_d \rightarrow e^\pm \tau^\mp$            | $2.1 \times 10^{-10}$           | $2.8 \times 10^{-5}$             |
| $B_s \rightarrow e^\pm \tau^\mp$            | $1.9 \times 10^{-10}$           | —                                |
| $B_d \rightarrow \mu^\pm \tau^\mp$          | $2.1 \times 10^{-10}$           | $2.2 \times 10^{-5}$             |
| $B_s \rightarrow \mu^\pm \tau^\mp$          | $1.9 \times 10^{-10}$           | —                                |

TABLE I: Maximal values of the branching ratios for LFV decays in the FCNC mediated  $Z$  boson model, after imposing the constraints on  $\text{Br}(\mu^- \rightarrow e^- e^+ e^-)$ ,  $\tau^- \rightarrow e^- \mu^+ \mu^-$  and  $\tau^- \rightarrow \mu^+ \mu^- \mu^-$ .

## VI. CONCLUSION

We have studied LFV decays of  $\mu$ ,  $\tau$  and  $B_{d,s}$  mesons in the model with additional vector-like leptons. In such a model due to the mixing between the exotic singlet leptons with the SM leptons, flavor changing neutral current transitions can occur at the tree level mediated by  $Z$  boson. Due to such couplings, the LFV decay modes considered here  $l_i \rightarrow l_j l_k \bar{l}_k$ ,  $B_{d,s} \rightarrow l_i \bar{l}_j$ ,  $\tau \rightarrow \mu \phi$  and the  $\mu$ - $e$  conversion in nuclei can arise at the tree level in this model. Assuming that the SM contributions to such decay modes have negligible effect we obtain the branching ratios for these LFV modes. The constraint on the new physics parameters are obtained using the present experimental limits on  $\mu^- \rightarrow e^- e^+ e^-$ ,  $\tau^- \rightarrow e^- \mu^+ \mu^-$  and  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ . These bounds impose strong constraints on the branching ratios of the other LFV decays. In Table-1, we present the upper limits of the branching ratios of various LFV decay modes using these constraints. We find that the LFV decays involving a  $\tau$  meson i.e.,  $\text{Br}(B_{d,s} \rightarrow \tau^\pm \mu^\mp, \tau^\pm e^\mp) \sim \mathcal{O}(10^{-10})$ , which could be observed in the upcoming high

sensitivity experiments. However, analogous LFV decays such as  $B_{d,s} \rightarrow \mu e$ , have branching ratios of the order of  $\mathcal{O}(10^{-18})$ , and are too small to be observed in the near future. We also find that the branching ratio of the semileptonic decay of  $\tau$  meson  $\tau \rightarrow \mu \phi$  could be as large as  $\mathcal{O}(10^{-8})$  in this model, the observation of which would unambiguously point to the presence of new physics. However, the branching ratio for  $\mu \rightarrow e \gamma$  is found to be quite small in this model as it occurs at one-loop level.

### Acknowledgments

The author would like to thank Department of Science and Technology, Government of India, for financial support through Grant No. SR/S2/RFPS-03/2006.

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